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COMPREHENSIVE UNIVERSITY STUDY NOTES

COMPUTER VISION

Unit II — Image Enhancement and Spatial Domain Filtering

Topics Covered in this Unit

Image Enhancement Basics: Objective vs Subjective Enhancement, Point Operations & Histogram Mapping
Spatial Domain Filtering Foundations: 2D Correlation vs Convolution Mechanics, Padding & Boundary Handling
Linear Smoothing Spatial Filters: Mean (Box) Filtering, Weighted Averaging & Gaussian Smoothing Filter
Non-Linear Spatial Filters: Order-Statistic Filters, Median Filtering & Bilateral Edge-Preserving Filter
Image Sharpening in Spatial Domain: First-Order Gradient Derivatives & Second-Order Laplacian Operators
Sharpening Enhancements: Unsharp Masking, High-Boost Filtering & Composite Laplacian Enhancement
Mathematical Noise Models: Gaussian Noise (Additive White), Salt-and-Pepper (Impulse/Bipolar) Noise
Specialized Noise Formulations: Poisson (Shot/Quantum) Noise & Speckle (Multiplicative) Ultrasound Noise
Comparative Filtering Strategies for Noise Removal & Preservation of Fine Structural Detail
High-Yield University Exam Question Bank with Detailed Model Answers & Unit Summary

Compiled in a structured, easy-to-understand, textbook style format
Specially curated for B.Tech / B.E. / BCA / MCA / B.Sc Computer Science, AI & Data Science Students

COURSE SYLLABUS & MAPPING

[UNIT II SYLLABUS — PRESCRIBED UNIVERSITY CURRICULUM]

This section contains the official syllabus for Unit II. Verify with your university curriculum mapping.

Unit II Syllabus Breakdown:

Unit II Image Enhancement and Spatial Domain Filtering: Image Enhancement Basics: Point operations & intensity transformations, Histogram equalization & specification. Spatial Domain Filtering: Convolution concept, Smoothing filters: Mean, Gaussian, Median, Bilateral Sharpening filters. Noise models and basic noise removal: Types of Noise Models, Gaussian Noise, Salt-and-Pepper Noise, Poisson Noise (Shot Noise), Speckle Noise – multiplicative noise (common in medical/ultrasound images)

• Course Code: ____________________ • Semester / Year: ____________________

TABLE OF CONTENTS

2.1 Image Enhancement Basics & Spatial Domain Concept

Definition: Image Enhancement in the Spatial Domain

Image enhancement encompasses computational techniques that process an input image so that the result is more suitable than the original image for a specific application (human visual interpretation or automated machine vision tasks). Spatial domain techniques operate directly on the 2D image pixels:

$g(x,y) = T[\, f(x,y)\,]$, *where T is an operator defined over a spatial neighborhood of (x,y).*

Point Operations vs. Neighborhood Spatial Filtering

- • **Point Operations (1×1 Neighborhood):** The transformed pixel intensity at coordinate (x,y) depends only on the single input pixel value at (x,y). Examples include Contrast Stretching, Intensity Thresholding, Negative transformations, Log/Power-law mappings, and Histogram Equalization.
- • **Neighborhood Operations (Spatial Mask / Kernel Filtering):** The transformed pixel intensity g(x,y) is computed by combining input pixel values within a defined local rectangular neighborhood (e.g., 3×3, 5×5, 7×7 window) centered at (x,y) using a mathematical mask.

2.2 Spatial Domain Filtering: Concept of 2D Convolution

Mathematical Formulation: 2D Discrete Convolution

2D convolution between an image f(x,y) of size M × N and a spatial filter mask w(s,t) of odd size m × n (where a = (m-1)/2 and b = (n-1)/2) involves rotating the filter mask by 180 degrees and computing the sliding sum of products:

$$g(x,y) = w(x,y) * f(x,y) = \sum[s=-a \text{ to } a] \sum[t=-b \text{ to } b]\, w(s,t)\, f(x-s, y-t)$$

Correlation vs. Convolution

- • **2D Correlation:** Slides the spatial filter mask w(s,t) across the image without rotating the mask:

$$w(x,y) \star f(x,y) = \sum[s=-a \text{ to } a] \sum[t=-b \text{ to } b]\, w(s,t)\, f(x+s, y+t)$$

- • **Distinction & Practical Consequence:** Correlation is used for template matching (finding a sub-image pattern). Convolution is used for linear filtering (smoothing, sharpening). If the filter mask is symmetric (such as a Gaussian or Laplacian kernel), 2D correlation and 2D convolution yield mathematically identical outputs.

- • **Boundary Padding Techniques:** To handle border pixels where the filter mask hangs over the image edge:
  - **1. Zero Padding:** Surround image boundaries with zeros (causes dark border artifacts).
  - **2. Replicate (Clamping) Padding:** Extend nearest edge pixel values outwards.
  - **3. Reflect / Mirror Padding:** Mirror pixel intensities across image borders (preserves gradient continuity).

2.3 Smoothing Spatial Filters (Linear & Non-Linear)

Smoothing filters are used for blurring and noise reduction. Blurring is commonly used in preprocessing tasks, such as removing small details from an image prior to large-scale object extraction and bridging small gaps in lines or curves:

1. Mean (Averaging / Box) Filter

- • **Mathematical Mechanism:** Replaces the center pixel value with the arithmetic mean of all pixel intensities in the $m \times n$ filter neighborhood. For a standard 3×3 box filter:

$$w = (1/9) \times [[1, 1, 1], [1, 1, 1], [1, 1, 1]]$$

- • **Characteristics:** Fastest linear smoothing filter; effectively reduces random white noise, but causes significant edge blurring and loss of high-frequency texture details.

2. Gaussian Smoothing Filter

- • **Mathematical Formulation:** A linear filter whose weights are sampled from an isotropic 2D Gaussian distribution centered at origin with standard deviation σ:

$$G(x,y) = (1 / (2 \pi \sigma^2)) \times \exp(- (x^2 + y^2) / (2 \sigma^2))$$

- • **Standard 3×3 Discrete Integer Approximation:** $w = (1/16) \times [[1, 2, 1], [2, 4, 2], [1, 2, 1]]$
- • **Key Properties:** Rotationally symmetric, separable filter ($G(x,y) = G(x) \times G(y)$), which reduces 2D convolution computational complexity from $O(M \times N \times K^2)$ to $O(2 \times M \times N \times K)$. Produces much smoother, artifact-free blurring than box filters.

3. Median Filter (Non-Linear Order-Statistic Filter)

- • **Mathematical Mechanism:** Ranks all pixel intensity values in the neighborhood in ascending order and replaces the center pixel with the median (50th percentile) value:

$$g(x,y) = \mathrm{median}\{ f(x+s, y+t) \mid (s,t) \in \mathrm{Neighborhood} \}$$

- • **Superiority over Linear Filters:** Extremely effective at removing Salt-and-Pepper (impulse) noise without blurring sharp edge boundaries. Because the median value is an actual sampled pixel value from the neighborhood, extreme outlier spikes (0 or 255) are completely eliminated rather than averaged in.

4. Bilateral Filter (Edge-Preserving Non-Linear Smoothing)

> **Definition: Bilateral Filter**
> *The Bilateral Filter is an advanced non-linear spatial filtering technique that smooths images and removes noise while strictly preserving sharp structural edges. It achieves this by combining a spatial domain Gaussian weight (geometric closeness) with a radiometric range Gaussian weight (photometric pixel similarity).*

- • **Mathematical Formula:**

$$g(x,y) = (1 / W_p) \sum_{[q \in \Omega]} f(q) \times G_{\sigma s}(||p - q||) \times G_{\sigma r}(|f(p) - f(q)|)$$

- • **Dual Weight Breakdown:**
  - **1. Spatial Kernel $G_{\sigma s}$:** Weights pixels inversely proportional to spatial Euclidean distance between center pixel p and neighbor q.
  - **2. Range Kernel $G_{\sigma r}$:** Weights pixels inversely proportional to radiometric intensity difference $|f(p) - f(q)|$. When the filter crosses an edge, neighboring pixels on the other side of the edge have large intensity differences, causing their range weights to drop to near zero, preventing edge blurring.

Filter Type	Linearity & Category	Primary Mathematical Function	Optimal Workload & Edge Behavior
Mean (Box) Filter	Linear Smoothing	Uniform average of neighborhood pixels.	Broad general noise attenuation; heavily blurs edges and corners.
Gaussian Filter	Linear Smoothing	Distance-weighted Gaussian bell curve.	Optimal removal of additive Gaussian noise; smooths high-frequency gradients.
Median Filter	Non-Linear Order-Statistic	Statistical median of sorted pixel values.	Gold standard for Salt-and-Pepper impulse noise; preserves step edges.
Bilateral Filter	Non-Linear Edge-Preserving	Spatial Gaussian × Range Intensity Gaussian.	Denoising and photography beautification without blurring object edges.

2.4 Sharpening Spatial Filters (Derivatives & High-Boost)

The principal objective of sharpening is to highlight transitions in intensity (edges, contours, and fine details). Image differentiation enhances edges and discontinuities while de-emphasizing regions of slowly varying gray levels:

1. First-Order Derivatives (The Gradient ∇f)

- • **Mathematical Gradient Vector:** $\nabla f = [\partial f/\partial x, \partial f/\partial y]^T = [g_x, g_y]^T$
- • **Gradient Magnitude:** $M(x,y) = \sqrt{g_x^2 + g_y^2} \approx |g_x| + |g_y|$
- • **Gradient Direction:** $\alpha(x,y) = \arctan(g_y / g_x)$ (perpendicular to edge direction).
- • **Property:** Produces thicker edges than second-order derivatives; non-zero response along gray-level ramps and step edges.

2. Second-Order Derivative: The Laplacian ($\nabla^2 f$)

- • **Definition:** The Laplacian is a linear, isotropic (rotation-invariant) 2D second-order differential operator:

$$\nabla^2 f = (\partial^2 f / \partial x^2) + (\partial^2 f / \partial y^2) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

- • **Standard 3×3 Discrete Laplacian Kernels:**

Kernel 1 (4-connectivity): [[0, 1, 0], [1, -4, 1], [0, 1, 0]] Kernel 2 (8-connectivity): [[1, 1, 1], [1, -8, 1], [1, 1, 1]]

- • **Composite Laplacian Image Sharpening:**

$g(x,y) = f(x,y) - \nabla^2 f(x,y)$ *[if center coefficient is negative]* $g(x,y) = f(x,y) + \nabla^2 f(x,y)$ *[if center coefficient is positive]*

3. Unsharp Masking and High-Boost Filtering

- • **Unsharp Masking 3-Step Process:**
  - 1. Blur the original image: $f_{smooth}(x,y) = \text{GaussianFilter}(f(x,y))$.
  - 2. Subtract blurred image from original to obtain the unsharp mask: $g_{mask}(x,y) = f(x,y) - f_{smooth}(x,y)$.
  - 3. Add weighted mask back to original: $g(x,y) = f(x,y) + k \times g_{mask}(x,y)$.
- • **High-Boost Filtering:** When $k = 1$, it is standard Unsharp Masking. When $k > 1$, it is High-Boost Filtering, which boosts high-frequency edge contrast while maintaining overall background brightness.

2.5 Mathematical Noise Models & Noise Removal Strategies

Mathematical Formulation of Image Noise

Noise is an undesirable electrical or optical disturbance that degrades image quality. In the spatial domain, degraded image g(x,y) is modeled as:

Additive Noise Model: $g(x,y) = f(x,y) + \eta(x,y)$

Multiplicative Noise Model: $g(x,y) = f(x,y) \times \eta(x,y)$

Taxonomy of Standard Image Noise Models

- **1. Gaussian Noise (Additive White Gaussian Noise - AWGN):** Arises due to electronic circuit thermal noise and sensor amplifier noise during low-light capture. Probability Density Function (PDF) follows a normal distribution:

$$p(z) = (1 / (\sqrt{(2\pi)}\, \sigma)) \times \exp(- (z - \mu)^2 / (2 \sigma^2))$$

- • **Best Removal Filter:** Arithmetic Mean Filter or Gaussian Filter.
- **2. Salt-and-Pepper Noise (Impulse / Bipolar Noise):** Caused by malfunctioning sensor pixels, faulty memory cells, or bit transmission errors over noisy channels. Pixels randomly take either maximum intensity (255 - Salt / white) or minimum intensity (0 - Pepper / black):

$$p(z) = P_a \text{ for } z = a \text{ (Pepper)},\quad p(z) = P_b \text{ for } z = b \text{ (Salt)},\quad p(z) = 0 \text{ otherwise}$$

- • **Best Removal Filter:** Median Filter or Adaptive Median Filter.
- **3. Poisson Noise (Shot Noise / Photon Noise):** Originates from the quantum physics nature of light, where photon arrival at sensor photodetectors follows a Poisson statistical process. Common in low-light night photography and medical X-ray imaging:

$$p(k) = (\lambda^k \times e^{(-\lambda)}) / k!, \quad \text{where } \lambda \text{ is the expected photon count}$$

- • **Best Removal Filter:** Anscombe Variance-Stabilizing Transform followed by Gaussian Denoising.
- **4. Speckle Noise (Multiplicative Noise):** A granular multiplicative interference pattern caused by coherent wave reflections interfering constructively and destructively. Heavily degrades Ultrasound imaging, Synthetic Aperture Radar (SAR), and Optical Coherence Tomography (OCT):

$$g(x,y) = f(x,y) \times \eta(x,y), \quad \text{where } \eta \text{ follows a Gamma or Rayleigh distribution}$$

- • **Best Removal Filter:** Lee Filter, Frost Filter, or Log-transform Homomorphic Filtering.

Noise Type	Statistical Model & Underlying Physical Cause	Mathematical Formulation	Optimal Restoration Filter
Gaussian Noise	Thermal amplifier electronic noise; zero-mean bell curve.	$p(z) = (1/\sqrt{(2\pi)}\sigma) \exp(-(z-\mu)^2 / 2\sigma^2)$	Gaussian Filter / Wiener Filter
Salt-and-Pepper Noise	Bit transmission error, dead sensor photo-sites.	Bipolar discrete probabilities at 0 and 255.	Median Filter / Adaptive Median
Poisson (Shot) Noise	Quantum photon arrival counting uncertainty.	$p(k) = (\lambda^k e^{(-\lambda)}) / k!$	Anscombe Transform + BM3D
Speckle Noise	Coherent wave interference in Ultrasound & SAR.	Multiplicative model: $g = f \times \eta$	Lee Filter / Homomorphic Denoising

2.6 High-Yield University Exam Question Bank & Model Answers

Part A: Short Answer Questions (2–3 Marks)

- **Q1. What is 2D Convolution in image processing? How does it differ from Correlation?**

Answer: 2D Convolution is a linear spatial filtering operation where the filter mask is rotated by 180 degrees before computing the sum of products with overlapping image pixels. 2D Correlation performs the sum of products directly without mask rotation. For symmetric filter masks (e.g., Gaussian, Laplacian), convolution and correlation are identical.

- **Q2. Why is the Median Filter non-linear, and why is it superior for Salt-and-Pepper noise?**

Answer: The Median Filter is non-linear because it does not satisfy the superposition property ($f(a+b) \neq f(a) + f(b)$). It sorts neighborhood pixels and picks the median value, which completely eliminates extreme impulse spikes (0 or 255) without averaging them into neighboring pixels, thereby preserving sharp edge transitions.

- **Q3. State the Laplacian operator formula and explain Composite Laplacian Sharpening.**

Answer: The Laplacian is a second-order isotropic derivative: $\nabla^2 f = \partial^2 f/\partial x^2 + \partial^2 f/\partial y^2$. In composite sharpening, the Laplacian is subtracted from the original image ($g = f - \nabla^2 f$ when center weight is negative), boosting fine high-frequency edge gradients while preserving low-frequency background brightness.

- **Q4. Explain Unsharp Masking and High-Boost Filtering.**

Answer: Unsharp masking subtracts a smoothed version of an image from itself to generate an edge mask ($g_{mask} = f - f_{smooth}$) and adds it back: $g = f + k \times g_{mask}$. When $k = 1$, it is unsharp masking. When $k > 1$, it is High-Boost filtering, which further amplifies edge detail.

Part B: Long Answer Questions (8–10 Marks)

- **Q5. Compare Mean, Gaussian, Median, and Bilateral filters in detail. Explain the mathematical working of the Bilateral Filter and why it preserves edges.**

Model Answer Outline:

- 1. Introduction to spatial smoothing and noise attenuation.
- 2. Analysis of Mean Filter (Box kernel, blurring side-effects) and Gaussian Filter (separable isotropic kernel, natural smooth blurring).
- 3. Analysis of Median Filter (Order-statistics, rank sorting, impulse noise immunity).
- 4. Comprehensive Bilateral Filter breakdown:
- - Formula: $g(p) = (1/W_p) \sum f(q)\, G_{\sigma s}(||p-q||)\, G_{\sigma r}(|f(p)-f(q)|)$.
- - Role of Spatial Gaussian $G_{\sigma s}$ (distance penalty).
- - Role of Range Gaussian $G_{\sigma r}$ (photometric intensity difference penalty).
- - Graphical illustration showing how weights collapse across step edges to prevent cross-edge blurring.
- 5. Detailed comparison matrix of computational complexity, filter linearity, and edge sharpness preservation.

- **Q6. Discuss the major Noise Models in Digital Images (Gaussian, Salt-and-Pepper, Poisson, Speckle). Describe the physical origin and optimal filtering technique for each.**

Model Answer Outline:

- 1. General mathematical degradation model: Additive ($g = f + \eta$) vs Multiplicative ($g = f \times \eta$).
- 2. Gaussian Noise: Normal PDF, thermal/sensor origin, removal via Gaussian/Averaging filters.
- 3. Salt-and-Pepper Noise: Bipolar impulse PDF, transmission bit-flips/dead pixels, removal via Median filtering.
- 4. Poisson Noise: Quantum photon counting process, low-light/X-ray origin, removal via Anscombe transform + Gaussian filtering.
- 5. Speckle Noise: Coherent wave interference in Ultrasound/SAR, multiplicative Rayleigh/Gamma distribution, removal via Lee/Homomorphic filters.
- 6. Summary table comparing PDFs, typical histogram shapes, and best-in-class spatial filters.

2.7 Unit II Summary & Quick Revision Sheet

Core Concept	Key Theoretical & Mathematical Takeaways for Exams
Spatial Convolution	$g(x,y) = \sum \sum w(s,t) f(x-s, y-t)$. Mask is flipped 180°. Symmetric masks make convolution = correlation.
Linear Smoothing	Mean filter (1/9 matrix, heavy blur), Gaussian filter (separable $e^{-(x^2+y^2)/2\sigma^2}$, natural smoothing).
Non-Linear Filters	Median filter (sorts pixels, immune to salt & pepper noise), Bilateral filter (Spatial Gaussian × Range Gaussian; preserves edges).
Sharpening Operators	First-order Gradient $\|\nabla f\| = \|g_x\| + \|g_y\|$; Second-order isotropic Laplacian $\nabla^2 f = f(x+1)+f(x-1)+f(y+1)+f(y-1)-4f$. Composite $g = f - \nabla^2 f$.
Noise Models	Gaussian (Additive electronic thermal noise), Salt-and-Pepper (Bipolar impulse), Poisson (Photon shot noise), Speckle (Multiplicative ultrasound/SAR noise).