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COMPREHENSIVE UNIVERSITY STUDY NOTES

ARTIFICIAL INTELLIGENCE

Unit IV — Knowledge, Logic and Reasoning

Topics Covered in this Unit

Knowledge-Based Agents: Knowledge Base (KB), Inference Engine, TELL & ASK Operations
The Wumpus World Problem: PEAS Specification, Formal Rules, Logical Percepts & Deductive Reasoning
Propositional Logic (PL): Syntax, Semantics, Truth Tables, Logical Connectives, Entailment & Equivalence
Propositional Theorem Proving: Inference Rules (Modus Ponens), Conjunctive Normal Form (CNF) & Resolution
Effective Model Checking: The DPLL Algorithm (Early Termination, Pure Symbols, Unit Clauses) & WalkSAT
First-Order Logic (FOL): Limitations of PL, Syntax & Semantics (Constants, Predicates, Functions, Quantifiers)
Knowledge Engineering in FOL: Domain Conceptualization, Vocabulary & Axiomatization
Inference in FOL: Universal & Existential Instantiation, Generalized Modus Ponens (GMP) & Unification (MGU)
Forward Chaining (Data-Driven / Datalog) vs. Backward Chaining (Goal-Driven / Prolog Logic Programming)
First-Order Resolution Refutation & Knowledge Representation (Ontologies, Semantic Nets, Frames & Inheritance)
High-Yield University Exam Question Bank with Detailed Model Answers & Unit Summary

Compiled in a structured, easy-to-understand, textbook style format
Specially curated for B.Tech / B.E. / BCA / MCA / B.Sc Computer Science, AI & Data Science Students

COURSE SYLLABUS & MAPPING

[UNIT IV SYLLABUS — PRESCRIBED UNIVERSITY CURRICULUM]

This section contains the official syllabus for Unit IV. Verify with your university curriculum mapping.

Unit IV Syllabus Breakdown:

IV Knowledge and Reasoning: Logical Agents, Knowledge-Based Agents, The Wumpus World problem, Propositional Logic: A Very Simple Logic, Propositional Theorem Proving, Effective Propositional Model Checking, Agents Based on Propositional Logic, First-Order Logic, Representation Revisited, Syntax and Semantics of First-Order Logic, Using First-Order Logic, Knowledge Engineering in First-Order Logic. Inference in First-Order Logic, Propositional vs. First-Order Inference, Unification and First-Order Inference, Forward Chaining, Backward Chaining, Resolution, Knowledge Representation

• Course Code: ____________________ • Semester / Year: ____________________

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4.1 Knowledge-Based Agents & The Wumpus World Problem

Definition: Knowledge-Based Agent (KBA)

A Knowledge-Based Agent is an intelligent agent whose central component is a Knowledge Base (KB) containing a set of sentences expressed in a formal Knowledge Representation Language. The agent interacts with its KB via two core interfaces:

• TELL(KB, sentence): Adds new background facts or percept assertions to the KB.

• ASK(KB, query): Queries the KB, returning answers derived via an Inference Engine.

The Wumpus World Benchmark Environment

The Wumpus World is a classic grid-based cave environment (typically 4×4) used to illustrate logical reasoning in partially observable domains:

PEAS Dimension	Specification in Wumpus World
Performance Measure	+1000 for climbing out of cave with Gold; -1000 for falling into a Pit or being eaten by Wumpus; -1 per action; -10 for shooting arrow.
Environment	4×4 grid of rooms. Start at [1,1] facing East. Gold and Wumpus placed randomly. Pits placed with 0.2 probability in any room except [1,1].
Actuators	Move Forward, Turn Left 90°, Turn Right 90°, Grab (gold), Shoot (arrow in facing direction), Climb (out of cave at [1,1]).
Sensors	5 discrete percepts: [Stench, Breeze, Glitter, Bump, Scream] (e.g., Stench in rooms adjacent to Wumpus; Breeze adjacent to Pits; Glitter in room with Gold; Scream when Wumpus dies).

Logical Reasoning Example in Wumpus World

- • **Fact 1 (Percept at [1,1]):** [None, None, None, None, None] → [1,1] is Safe. Rooms [1,2] and [2,1] are Safe (No Pit, No Wumpus).
- • **Fact 2 (Move to [2,1]):** Percept = [None, Breeze, None, None, None] → Pit is in either [2,2] or [3,1]. Agent turns back to [1,1] and moves to [1,2].
- • **Fact 3 (Percept at [1,2]):** Percept = [Stench, None, None, None, None] → Wumpus is in [1,3] or [2,2]. Since there was no Stench at [2,1], Wumpus CANNOT be in [2,2]; therefore, Wumpus is definitely in [1,3]! Furthermore, since there is no Breeze at [1,2], [2,2] has NO Pit! By deductive elimination, the Pit at [2,1] MUST be in [3,1]!

4.2 Propositional Logic: Syntax, Semantics & Theorem Proving

1. Syntax & Semantics

- • **Propositional Symbols:** Atomic sentences represented by uppercase letters (e.g., P, Q, W_1,3, B_2,1) representing true/false facts.
- • **Complex Sentences & Logical Connectives:**
  - • **Negation (¬):** ¬ P (NOT P)
  - • **Conjunction (∧):** P ∧ Q (P AND Q)
  - • **Disjunction (∨):** P ∨ Q (P OR Q)
  - • **Implication (→):** P → Q (IF P THEN Q; equivalent to ¬ P ∨ Q)
  - • **Biconditional (↔):** P ↔ Q (P IF AND ONLY IF Q; equivalent to (P → Q) ∧ (Q → P))

2. Entailment, Soundness & Completeness

Definition: Entailment (α ⊨ β)

Knowledge base KB entails sentence α (written KB ⊨ α) if and only if in every possible model (world state) where KB is True, α is also True (i.e., M(KB) ⊆ M(α)).

• Soundness: An inference algorithm derives only entailed sentences (no false derivations).

• Completeness: An inference algorithm derives every sentence that is entailed.

3. Propositional Resolution Algorithm (Refutation Complete)

- • **Principle of Proof by Contradiction:** To prove KB ⊨ α, assert (KB ∧ ¬α) and show it produces a Contradiction (Empty Clause □).
- • **Conjunctive Normal Form (CNF):** A conjunction of disjunctions of literals (e.g., (A ∨ ¬ B) ∧ (C ∨ D ∨ ¬ E)).
- • **Resolution Rule:** Given clauses (l_1 ∨ ... ∨ l_k ∨ P) and (m_1 ∨ ... ∨ m_n ∨ ¬ P), resolve complementary literals P and ¬ P to produce Resolvent:

(l_1 ∨... ∨l_k ∨m_1 ∨... ∨m_n)

4. Effective Model Checking: DPLL Algorithm (Davis-Putnam-Logemann-Loveland)

DPLL is a highly optimized recursive backtracking algorithm for Boolean Satisfiability (SAT) that incorporates 3 core optimizations:

- **1. Early Termination:** Detects if any clause is already satisfied (True) or if a whole clause is False under partial assignment.
- **2. Pure Symbol Heuristic:** A symbol that appears with the same sign (only positive or only negative) in all unsatisfied clauses is immediately assigned that truth value.
- **3. Unit Clause Heuristic:** A clause with only one unassigned literal must assign that literal the value that satisfies the clause (Unit Propagation cascade).

4.3 First-Order Logic (FOL): Syntax, Semantics & Quantifiers

Why First-Order Logic?

Propositional Logic lacks expressive power (cannot state general rules like 'All men are mortal' without writing infinite separate propositions for every individual). First-Order Logic (FOL) represents the world using Objects, Relations (Predicates), Functions, and Quantifiers.

Syntactic Elements of FOL

FOL Component	Grammatical Definition	Concrete Examples
Constants	Names referring to specific individual objects in the domain.	John, Mary, 2, Paris, KingArthur.
Variables	Placeholders ranging over objects in the domain.	x, y, z, person, country.
Predicates	Relations mapping tuples of objects to True/False.	Brother(x, y), King(John), GreaterThan(5, 3).
Functions	Mappings from tuples of objects to a single unique object.	FatherOf(John), Plus(2, 3), Sqrt(16).
Universal Quantifier (∀)	Universal generalization: True for ALL objects in domain.	∀x King(x) → Person(x) ('All kings are persons').
Existential Quantifier (∃)	Existential claim: True for AT LEAST ONE object in domain.	∃x King(x) ∧ Greedy(x) ('There exists a greedy king').

NOTE / EXAM POINTER

EXAM POINTER: Common quantifier translation errors: Typically, Universal Quantifier (∀) pairs with Implication (→), e.g., ∀x Bird(x) → Flies(x). Typically, Existential Quantifier (∃) pairs with Conjunction (∧), e.g., ∃x Bird(x) ∧ CannotFly(x).

4.4 Inference in First-Order Logic: Unification & Chaining

1. Unification & Most General Unifier (MGU)

Definition: Unification

Unification is the algorithmic procedure of finding a variable substitution θ = {v_1/t_1, v_2/t_2, ...} that makes two distinct logical expressions identical: UNIFY(p, q) = θ such that SUBST(θ, p) = SUBST(θ, q).

Expression 1 (p)	Expression 2 (q)	Most General Unifier (θ)	Unified Output
Knows(John, x)	Knows(John, Jane)	{x / Jane}	Knows(John, Jane)
Knows(John, x)	Knows(y, Bill)	{x / Bill, y / John}	Knows(John, Bill)
Knows(John, x)	Knows(y, MotherOf(y))	{y / John, x / MotherOf(John)}	Knows(John, MotherOf(John))
Knows(John, x)	Knows(x, Elizabeth)	FAIL (Occur Check: x cannot equal John and Elizabeth)	FAIL

2. Forward Chaining vs. Backward Chaining

Comparison Criteria	Forward Chaining (Data-Driven)	Backward Chaining (Goal-Driven)
Direction of Inference	Starts from known facts in KB and applies inference rules to derive new facts until goal is reached.	Starts from query goal and works backward through IF-THEN rules to find supporting ground facts.
Nature & Strategy	Data-Driven / Bottom-Up.	Goal-Driven / Top-Down (Depth-First Search).
Efficiency & Redundancy	May derive thousands of irrelevant facts unrelated to goal.	Focuses strictly on sub-goals relevant to proving query.
Applications & Engines	Production rule systems, Datalog, automated event monitoring.	Logic Programming (Prolog), automated theorem provers, expert diagnostic systems.

4.5 Knowledge Representation: Ontologies, Semantic Nets & Frames

- **1. Ontologies:** Formal explicit specifications of domain conceptualizations, defining taxonomic category hierarchies, object properties, and axioms.

- **2. Semantic Networks:** Graph representations where nodes represent concepts/objects and directed labeled edges represent semantic relationships (e.g., is-a, has-part, instance-of). Inherits properties via graph traversal.
- **3. Frames & Description Logics:** Record-like structures containing named slots and filler values (default values, inherited values, procedural triggers like if-needed/if-added). Form the foundation of modern Web Ontology Language (OWL) and Knowledge Graphs (Google Knowledge Graph).

4.6 High-Yield University Exam Question Bank & Model Answers

Part A: Short Answer Questions (2–3 Marks)

- **Q1. What is Unification in First-Order Logic? Give an example.**

Answer: Unification is an algorithmic process that finds a variable substitution θ making two logical expressions identical (UNIFY(p, q) = θ). Example: UNIFY(Knows(John, x), Knows(y, Mary)) yields substitution θ = {x / Mary, y / John}, producing unified sentence Knows(John, Mary).

- **Q2. Differentiate between Forward Chaining and Backward Chaining.**

Answer: Forward chaining is a data-driven, bottom-up inference strategy that starts with known facts and triggers rules to derive new facts. Backward chaining is a goal-driven, top-down strategy that starts from the query goal and searches backward to find ground facts supporting the hypothesis.

- **Q3. State the three heuristics used in the DPLL algorithm.**

Answer: (1) Early Termination (detecting satisfied or unsatisfied clauses early), (2) Pure Symbol Heuristic (assigning truth values to symbols appearing with uniform polarity), and (3) Unit Clause Heuristic (unit propagating clauses with only one unassigned literal).

- **Q4. Translate into First-Order Logic: 'Every student who studies hard passes the exam.'**

Answer: $\forall x\ (\ Student(x) \land StudiesHard(x)\ ) \rightarrow PassesExam(x)$.

Part B: Long Answer Questions (8–10 Marks)

- **Q5. Explain the Resolution Algorithm in Propositional Logic. Convert the following premises to CNF and prove the goal using Resolution Refutation.**

Model Answer Outline:

- 1. Resolution Principle: Sound and refutation-complete inference rule resolving complementary literals (P and ¬ P).
- 2. Step-by-step 6-stage CNF conversion procedure: Eliminate ↔, Eliminate →, Move ¬ inward (De Morgan's Laws), Distribute ∨ over ∧.

- 3. Resolution Refutation method: Negate target goal, add to CNF clause set, iteratively resolve clauses until Empty Clause □ (Contradiction) is derived.
- 4. Complete resolution refutation graph trace demonstrating derivation of □.

- **Q6. Describe the Wumpus World Environment. Detail its PEAS description, rules, and demonstrate how a Knowledge-Based Agent uses logical deduction to navigate safely.**

Model Answer Outline:

- 1. Formal PEAS description of Wumpus World (Performance measure, Grid environment, Actuators, Sensors).
- 2. Propositional Logic axiomatization: Stench rules $S_{x,y} \leftrightarrow (W_{x-1,y} \lor W_{x+1,y} \lor W_{x,y-1} \lor W_{x,y+1})$ and Breeze rules.
- 3. Complete step-by-step navigation walkthrough: Room [1,1] percepts → Move to [2,1] → Move to [1,2] → Deductive elimination proving exact coordinates of Wumpus and Pit.
- 4. Proof of safety and decision to grab gold and climb out.

4.7 Unit IV Summary & Quick Revision Sheet

Logic & Reasoning Domain	Key Theoretical & Formal Takeaways for Exams
Knowledge-Based Agents	KB + Inference Engine. TELL (store fact), ASK (query entailment $KB \vDash \alpha$).
Propositional Logic	Syntax (P, Q), Semantics (Truth tables). Resolution in CNF is Refutation Complete (derives □). DPLL model checker.
First-Order Logic (FOL)	Objects, Predicates, Functions, Quantifiers (∀ and ∃). Universal Instantiation & Skolemization.
FOL Inference	Unification UNIFY(p,q)=θ. Forward Chaining (Data-driven/Datalog) vs Backward Chaining (Goal-driven/Prolog).
Knowledge Representation	Ontologies (OWL), Semantic Networks (is-a links), Frames (Slots/Fillers), Knowledge Graphs.